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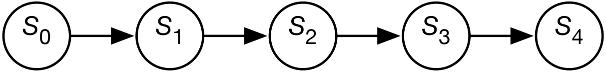

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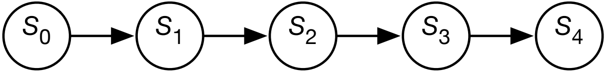

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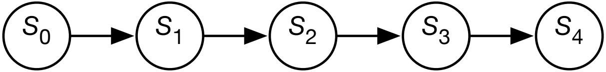

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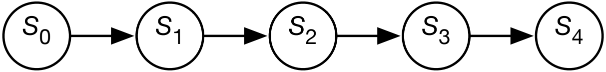

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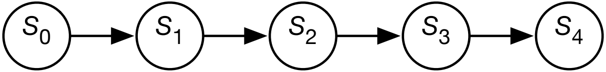

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$$S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4$$

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- Often S_t represents the state at time t. Intuitively S_t conveys all of the information about the history that can affect the future states.
- "The future is independent of the past given the present."

Stationary Markov chain

- A stationary Markov chain is when for all $i > 0$, $i' > 0$, $P(S_{i+1}|S_i) = P(S_{i'+1}|S_{i'})$.
- We specify $P(S_0)$ and $P(S_{i+1}|S_i)$.
  - Simple model, easy to specify
  - Often the natural model
  - The network can extend indefinitely

Stationary Distribution

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- An ergodic and aperiodic Markov chain has a unique stationary distribution P and $P(s) = \lim_{i\to\infty} P_i(s)$ — equilibrium distribution

Pagerank

Consider the Markov chain:

- Domain of S_i is the set of all web pages
- $P(S_0)$ is uniform; $P(S_0 = p_j) = 1/N$

$$
\begin{aligned}
& P(S_{i+1} = p_j \mid S_i = p_k) \\
& \quad = (1-d)/N + d * \begin{cases} 1/n_k & \text{if } p_k \text{ links to } p_j \\ 1/N & \text{if } p_k \text{ has no links} \\ 0 & \text{otherwise} \end{cases}
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where there are N web pages and n_k links from page p_k

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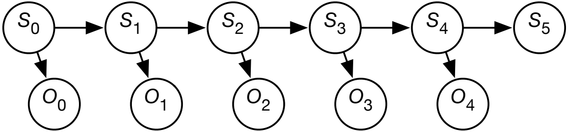
- $d \approx 0.85$ is the probability someone keeps surfing web
- This Markov chain converges to a distribution over web pages (original $P(S_i)$ for $i = 52$ for 322 million links): Pagerank - basis for Google's initial search engine

Hidden Markov Model

- A Hidden Markov Model (HMM) is a belief network:



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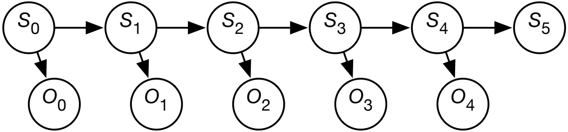

The probabilities that need to be specified:

- $P(S_0)$ specifies initial conditions
- $P(S_{i+1}|S_i)$ specifies the dynamics
- $P(O_i|S_i)$ specifies the sensor model

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$$P(S_i | o_1, \ldots, o_i)$$

What is the current belief state based on the observation history?

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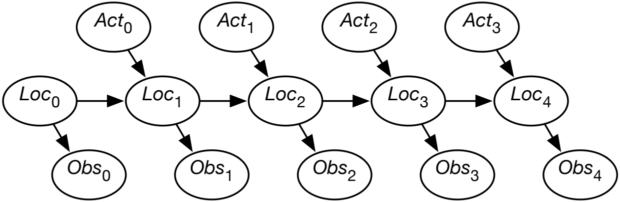
- then observe O_1, query S_1.
- then observe O_2, query S_2.
- ...

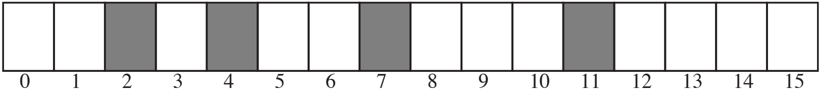
Example: localization

- Suppose a robot wants to determine its location based on its actions and its sensor readings: Localization
- This can be represented by the augmented HMM:



Example localization domain

- Circular corridor, with 16 locations:



- Doors at positions: 2, 4, 7, 11.
- Noisy Sensors
- Stochastic Dynamics
- Robot starts at an unknown location and must determine where it is.

Example Sensor Model

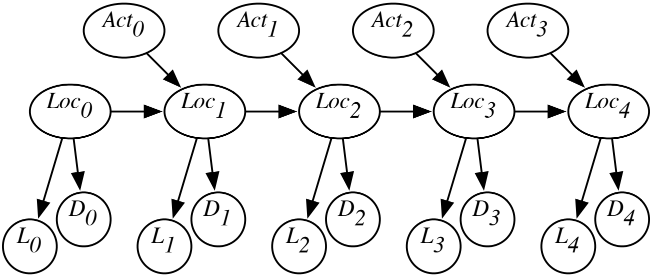
- $P(Observe\ Door \mid At\ Door) = 0.8$
- $P(Observe\ Door \mid Not\ At\ Door) = 0.1$

Example Dynamics Model

- $P(loc_{t+1} = L | action_t = goRight \land loc_t = L) = 0.1$
- $P(loc_{t+1} = L+1 | action_t = goRight \land loc_t = L) = 0.8$
- $P(loc_{t+1} = L+2 | action_t = goRight \land loc_t = L) = 0.074$
- $P(loc_{t+1} = L' | action_t = goRight \land loc_t = L) = 0.002$ for any other location L'.
  - All location arithmetic is modulo 16.
  - The action *goLeft* works the same but to the left.

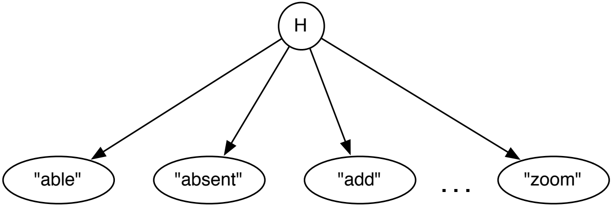
Combining sensor information

- Example: we can combine information from a light sensor and the door sensor Sensor Fusion



S_t robot location at time t
D_t door sensor value at time t
L_t light sensor value at time t

Naive Bayes Classifier: User's request for help



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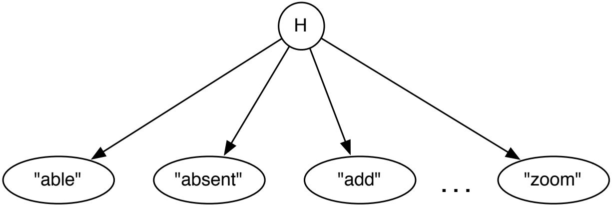

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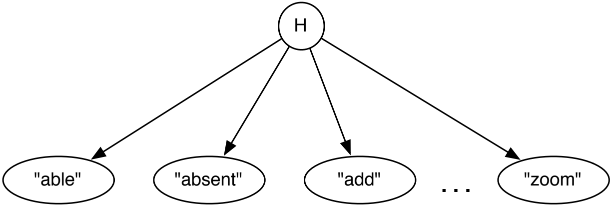

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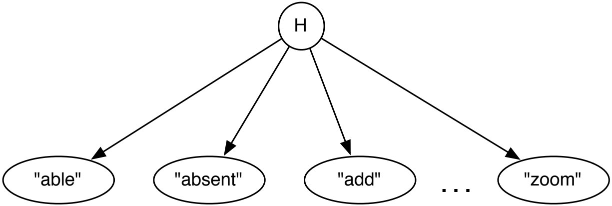

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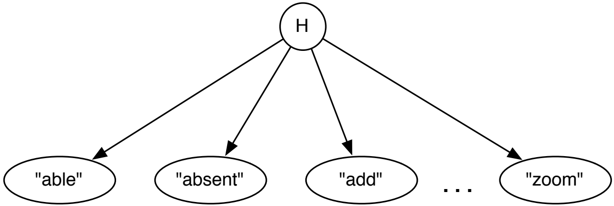

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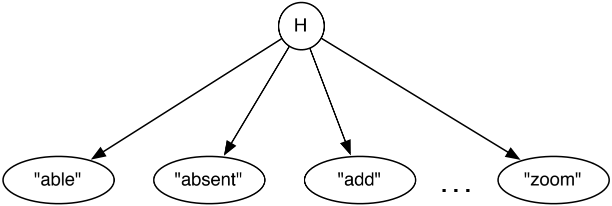

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- Given a help query: condition on the words in the query and display the most likely help page.

Sentence: $w_1, w_2, w_3, \ldots$.

Set-of-words model:

- Each variable is Boolean: *true* when word is in the sentence and *false* otherwise.

Simple Language Models: set-of-words

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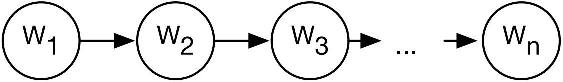

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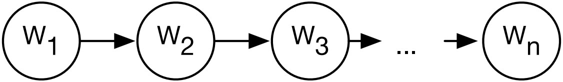

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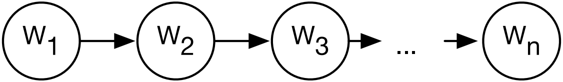

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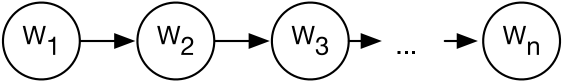

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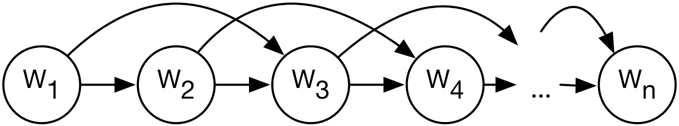

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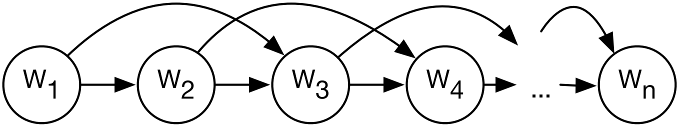

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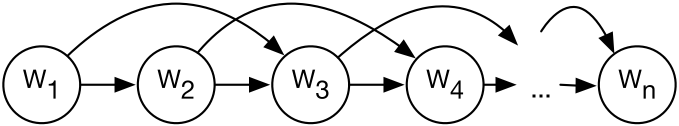

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N-gram

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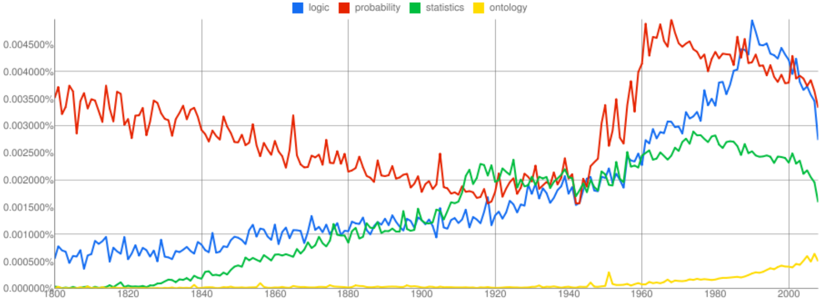
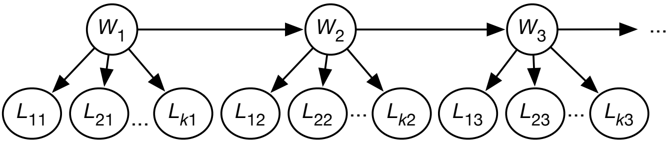
Logic, Probability, Statistics, Ontology over time



From: Google Books Ngram Viewer
(`https://books.google.com/ngrams`)

Predictive Typing and Error Correction



$$domain(W_i) = \{"a", "aarvark", \ldots, "zzz", "\bot", "?"\}$$
$$domain(L_{ji}) = \{"a", "b", "c", \ldots, "z", "1", "2", \ldots\}$$

Beyond N-grams

- *A person with a big hairy cat drank the cold milk.*
- Who or what drank the milk?

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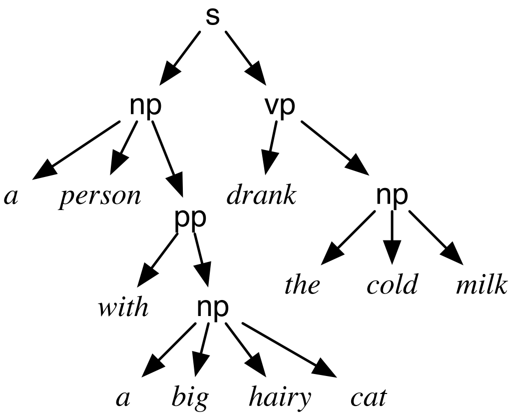
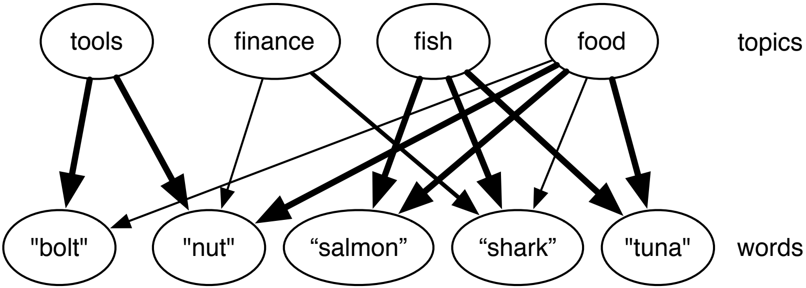
Simple syntax diagram:



Topic Model


tools
finance
fish
food
topics
"bolt"
"nut"
“salmon”
“shark”
"tuna"
words

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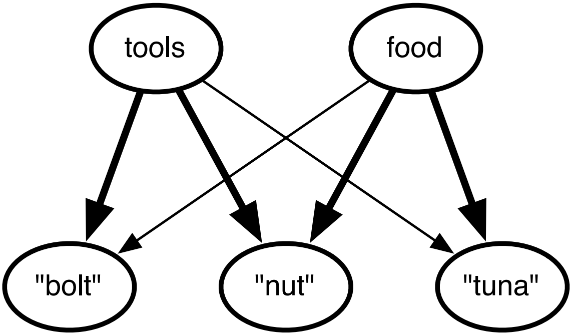
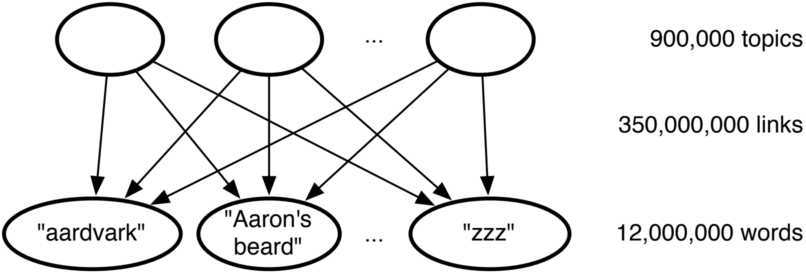


topics

words

Google's rephil



Deep Belief Networks